Azimuth Correction for Elevation-over-Azimuth Positioners

Earth stations equipped with elevation-over-azimuth (El/Az) positioners move the antenna boresight at zero elevation in azimuth on a great circle. The arc length of the travelled azimuth path is the spherical central angle in radians.

In El/Az positioners the elevation axis is mounted over the azimuth axis. When azimuth is changed the elevation axis also moves in the azimuth plane. On the other hand elevation changes totally independently without affecting azimuth.

When the antenna is pointed in elevation at non-zero angles, e.g. at a geostationary satellite, the El/Az mount moves the antenna boresight no longer on a great circle but on a small circle. The spherical arc length of the travelled azimuth path on the small circle is smaller than before.

As a consequence earth stations performing azimuth scans with non-zero elevation angles need to correct azimuth encoder readouts. The azimuth correction formula for El/Az positioners is:

$$\sin\left(Az_{\text{true angle}} / 2 \right) = \sin\left(Az_{\text{encoder}} / 2 \right) \cdot \cos\left(El \right)$$

$$Az_{\text{true angle}} = 2 \cdot \arcsin\left(\sin\left(Az_{\text{encoder}} / 2 \right) \cdot \cos\left(El \right) \right)$$

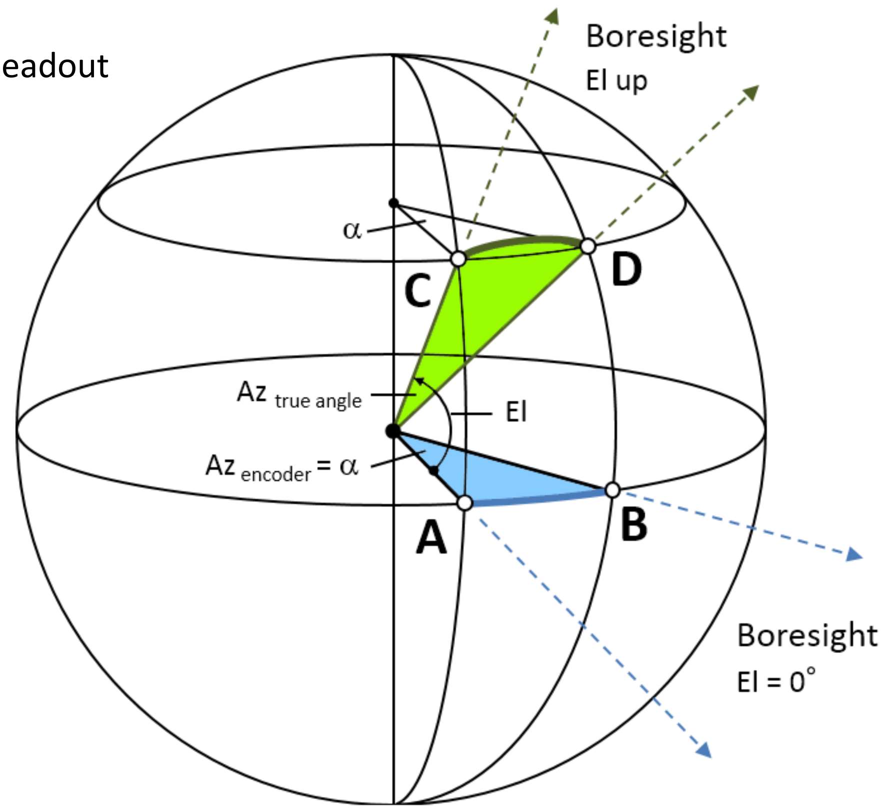
In this memo the azimuth correction formula is developed using trigonometry. The earth station is in the center of a spherical azimuth elevation coordinate system. The antenna boresight is scanned in azimuth along the equator of the sphere (this is encoder readout) and moved up or down in elevation with zero pointing along the plane of the equator.

Azimuth Encoder Readout

$\alpha = Az_{\text{encoder}}$

True Azimuth

$Az = Az_{\text{true angle}}$



Step 1

View top down on the sphere. The equator appears as a circle with radius R. The antenna boresight scanned in azimuth from position A to B with elevation at zero. The length of the travelled arc is the central angle (α) in radians. The central angle is identical with the spherical angle and consequently this is true azimuth.

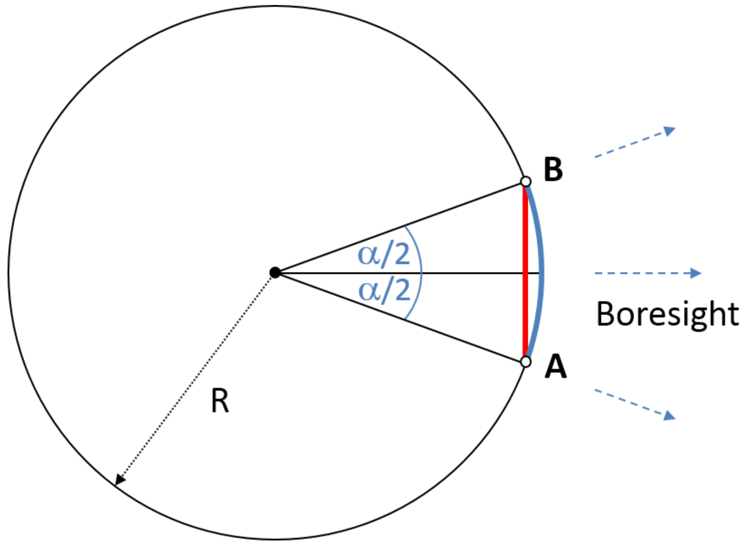
Arc length AB (blue line)

$\angle AB = R \cdot \alpha rad$

Segment AB (red line)

$AB = 2 \cdot R \cdot \sin(\alpha/2)$

R = sphere radius

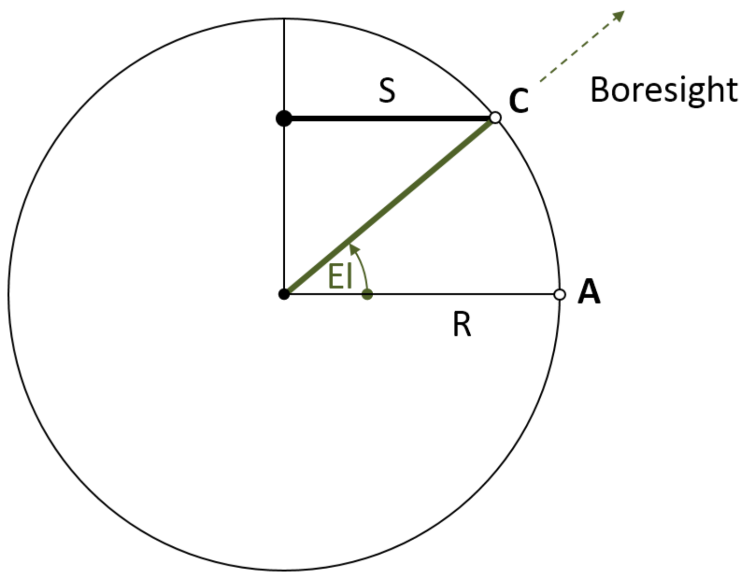


Step 2

View from the side. The equator appears as a straight line through the center of the sphere to position A. The antenna boresight is now pointed upwards in elevation at angle (El) and hits the sphere at position C. Line segment S is the radius of a small circle compared to radius R of the equator which is a great circle.

Half segment S (bold line)

$S = R \cdot \cos(El)$

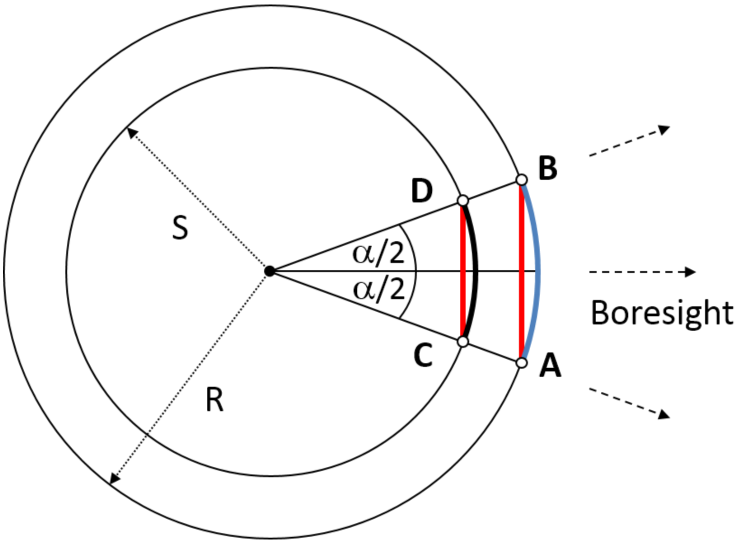


Step 3

View from top down again. The equator appears as a great circle as before. When scanning in azimuth from positions A to B with elevation pointed up at angle (El) the boresight hits the sphere from positions C to D. These two positions lie on a small circle with radius S. In this view the small circle appears as concentric circle inside the equator circle. The central angle (α) is the same; the arc length CD (bold black) is NOT a spherical arc length but the arc length of the small circle.

Segment CD (red line)

$CD = 2 \cdot S \cdot \sin(\alpha/2)$



Step 4

View of the sphere from a position perpendicular to the area of the triangle formed by positions C and D and the center of the sphere. The arc between positions C and D is the length of the spherical segment. The straight red line connecting positions C and D was calculated before. What needs to be calculated is the green spherical arc length. This is the value of the angle ($Az_{\text{true angle}}$) in radians – which is true azimuth.

True azimuth angle Az true angle

$\sin(Az/2) = S / R \cdot \sin(\alpha/2)$

$\sin(Az/2) = \sin(\alpha/2) \cdot \cos(El)$

$Az = 2 \cdot \arcsin(\sin(\alpha/2) \cdot \cos(El))$

$Az_{\text{true angle}} = Az$